# Curve-Growth Analysis by Using Micro-Computer II. Applications to Normal and Carbon Stars


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# Curve-of-Growth Analysis by Using a Micro-Computer

## II. Applications to Normal and Carbon Stars

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# マイクロコンピューターを用いた成長曲線解析法

## II．標準化学組成星と炭素星への適用

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要　旨

成長曲線解析法は，恒星大気の分光分析の主要な方法の1つである．この方法については，従来，経験成長曲線と理論成長曲線の一致を目測で行なっていたため，得られた結果に個人差があり，また誤差の客観的な見積りが困難であるという問題があった．これを解決するため，本研究年報第4号において，筆者は成長曲線解析法のマイコン用のプログラムを開発した．

本研究では，このプログラムを改良した．そして，すでに目測による成長曲線解析法で分光分析のなされている2つの星，すなわち標準化学組成の星である $\phi$ UMa と炭素星である $U$ Hya に対して，同じデータを用いて本プログラムに基づく解析を行ない，すでに得られている結果と比較した．それによれば，測定精度の高いデータをもつ $\phi$ UMa に対してはすでに得られているよりもむしろ一貫性のある結果が得られたが，測定精度の低いデータをもつ $U$ Hya に対しては，大きくくい違う結果が得られた．これらの結果は，精度の高いデータに対しては本プログラムは十分適用できることを示すとともに，精度の低いデータに対しては，本プログラムで採用されている解析過程の方針が適用不可能であることを示唆している．

## I. Introduction

A curve-of-growth analysis is one of the important methods which are used for an analysis of a stellar atmosphere. In this method, one-layer approximation is made, that is, it is assumed that the stellar atmosphere is a uniform one with a specific value for a physical quantity such as pressure, temperature and density. A curve-of-growth is used in this method. The curve-of growth is a graphical representation of the relation for absorption lines between the logarithm of the equivalent width divided by wavelengh log $W/\lambda$ and the logarithm of the number of absorbing atoms times the oscillator strength log $Nf$, where $f$ is the oscillator strength and the equivalent width $W$ is the width of the rectangular profile for which the height is equal to the continuum level near the line and the area is that of the line.

In the curve-of-growth, the following quantity usually is plotted along the abscissa $\log X_{ab}$ on the assumption of thermodynamic equilibrium,

$$\log X_{ab}=\log(gf\lambda)-\theta_{ex}\chi_1, \qquad (1)$$

where $g$ is the statistical weight of the lower energy level and $\chi_1$ is the excitation potential of the lower energy level; $\theta_{ex}$ is the reciprocal excitation temperature $5040/T_{ex}$ ($T_{ex}$ is the excitation temprature).

In cases in which the $\log(gf)$ values are not known, the following quantity is plotted along the abscissa instead of $\log X_{ab}$,

$$\log X_{rel}=\log X_s-\Delta\theta_{ex}\chi_1, \qquad (2)$$

where $\log X_s$ is the abscissa of the curve-of-growth for a standard star for which the physical quantities and chemical composition are already known; $\Delta\theta_{ex}$ is the difference in $\theta_{ex}$ between a relevant star and the standard star. In this case, the relative values to the standard star for physical quantities and the chemical composition are obtained instead of the absolute values. This method is called differential curve-of-growth analysis. On the other hand, the method where $\log X_{ab}$ is plotted as abscissa and the absolute values are obtained is called absolute curve-of-growth analysis.

Curve-of-growth analyses have conventionally been applied by eye measure. Thus, there has been a fear that the results by this method depend on the subjectivity of an analyzer. Moreover, an objective estimate of error can not been made for this method.

Recently, the curve-of-growth analyses by using a computer have been applied in order to overcome the above weak points (For example, Tech (1971)[1] and Powell (1971)[2]). However, these methods require a large amount of memory capacity and can be applied only to a large-sized computer. Yoshioka (1987)[3] (Hereafter it is referred to as Paper I) developed the curve-of-growth analysis by using a micro-computer. In Paper I, the merits of the procedures by Tech (1971) and by Powell (1971) were made use of and developed for use with a micro-computer PC-9801 (NEC). Two kinds of procedures were developed, which are called the method of type 1 and of type 2 respectively in Papar I, and the superiority of the method of type 2 has been indicated.

In this paper, the micro-computer program of the method of type 2 has been improved. This method has also been applied to two stars, $\psi$ UMa and $U$ Hya, using the same data as Yamashita (1967)[4] and Utsumi (1970)[5] respectively for $\psi$ UMa and $U$ Hya and the results have been compared with those of Yamashita (1967) and Utsumi (1970).

## II. The Procedure of the Method of Type 2

In curve-of-growth analyses, a theoretical curve-of-growth which is considerd to fit an empirical curve-of-growth is adopted, that is, the model of the stellar atmosphere, the value of damping constant and the the vertical shift between the theoretical curve-of-growth and the empirical curve-of-growth are adopted, where the empirical curve-of-growth means the whole group of points of lines in the curve-of-growth plane. Then, the value of $\theta_{ex}$ (or $\Delta\theta_{ex}$) is determined from the horizontal shift between the theoretical curve and the empirical curve by the expression (1) (or (2)).

In the method of type 2, the value of $\theta_{ex}$ (or $\Delta\theta_{ex}$) is determined simultaneously with the damping constant and the vertical shift, where the theoretical curve fitted to the empirical curve is that for pure absorption in the Milne-Eddington atmosphere calculated by Hunger (1956)[6]. For the Milne-Eddington atmosphere the vertical shift equals to $\log(c/2R_C V_D)$, where $c$ is the speed of light and $V_D$ is the Doppler velocity; $R_C$ is the limiting central depth for strong lines. The determinations are done in the following way. First, the value of damping parameter $\log 2\alpha$ $(2\alpha=\lambda\Gamma/2\pi V_D)$ is setted, where $\Gamma$ is the damping constant. Secondly, the value of $\theta_{ex}$ (or $\Delta\theta_{ex}$) is determined as least-squares solution in the direction parallel to the abscissa for various values of $\log(c/2R_C V_D)$. Thirdly, the $\log(c/2R_C V_D)$ value and the corresponding value of $\theta_{ex}$ (or $\Delta\theta_{ex}$) which give a minimum value of the standard deviation $\sigma_{temp}$ of the $\theta_{ex}$ (or $\Delta\theta_{ex}$) value are selected. The above process is repeated for various values of $\log 2\alpha$, and the $\log 2\alpha$ value and the corresponding values of $\theta_{ex}$ (or $\Delta\theta_{ex}$) and $\log(c/2R_C V_D)$ for which the $\alpha_{temp}$ value is minimum are adopted as the final values for these quantities. In the above process, a gradient of the theoretical curve-of-growth for the ordinate of a line is taken into account as a weight for a least-squares solution so that the lines on the linear and damping parts of the curve-of-growth are given heavier weight than those on the flat parts of the curve-of-growth. The program for the above procedure written in BASIC is named "COG3" and is listed in Appendix 2 of Paper I. This program has been improved in this paper on the following three points: 1) The range of the $\log 2\alpha$ value for the theoretical curve-of-growth has been extended so as to include the range from $-3.5$ to $-4$; 2) In the new program, the values of $\theta_{ex}$ (or $\Delta\theta_{ex}$) and $\log(c/2R_C V_D)$ for which the $\alpha_{temp}$ value is minimum are selected from the inputted range of the $\log(c/2R_C V_D)$ value for the specified $\log 2\alpha$ value, while in the old program, the values of $\theta_{ex}$ (or $\Delta\theta_{ex}$) and $\alpha_{temp}$ are computed for the specified values of $\log(c/2R_C V_D)$ and $\log 2\alpha$; 3) In the new program, the process can be repeated for other values of $\log 2\alpha$, while in the old program, it cannot be repeated. The new program also written in BASIC is named

" COGO " and it is listed in the Appendix.

## III. Application to $\psi$ UMa

The method has been applied to $\psi$ UMa with almost normal chemical composition to check the applicability of it to data with high accuracy.

### III.1. Curve-of-Growth Analysis of $\psi$ UMa by Yamashita

$\psi$ UMa (=HD96833) is a K1 type giant with $m_v=3.01$ according to Yale Catalogue of Bright Stars by Hoffleit (1964)[7]. According to Roman (1952)[8], this star belongs to the weak-line group, though both the radial velocity $V_r=-4$ km/s and the random velocity referred to the local standard of rest $V=11$ km/s are small, which values are taken from the above catalogue and Roman (1952), respectively.

Yamashita (1967) analyzed $\psi$ UMa and $\chi$ UMa (K0 III) together with 37 Com (K1 p) by an absolute curve-of-growth method, using spectrograms whose linear dispersions ranged from 3.4 Å/mm to 7.0 Å/mm. Four spectral regions were analyzed, that is, ultraviolet region (3750 Å~4100 Å), blue region (4150 Å~4500 Å), yellow region (5250 Å~6000 Å) and red region (6150 Å~6850 Å). He adopted the theoretical curve-of-growth by Hunger (1956) for pure absorption in the Milne-Eddington atmosphere. Moreover, he adopted the following quantity as the abscissae of the empirical curve-of-growth, instead of the quantity by the expression (1),

$$\log X_{ab}=\log (gf\lambda)-\theta_{ex}\chi_1-\log (\kappa/\kappa_{5000}), \qquad (3)$$

where $\kappa$ and $\kappa_{5000}$ are the continous absorption coefficient at the relevant wavelength and 5000 Å, respectively. This term was added in order to correct for the variation of the continuous absorption coefficient with wavelength.

The numbers of Fe and Ti atoms above the photosphere for $\psi$ UMa determined by Yamashita (1967) are slightly smaller than those for the Sun determined by Goldberg et al. (1960) by $-0.39$ dex and $-0.43$ dex, respectively. However, the above underabundances of Fe and Ti for $\psi$ UMa need not to be taken seriously, as stated by Yamashita (1967), because the depth of the photosphere for $\psi$ UMa differs from that for the Sun. Yamashita (1967) derived also the differences in the abundances of other elements between $\psi$ UMa and the Sun normalized in such a way that the mean abundance of Fe and Ti equals to zero. Then, Li, Mn, Zn, Ga, Rh and Sb are overabundant by a factor of more than 4 and Mg, Ni and Yb are underabundant by a factor of more than 4. The relative abundances of the other elements agree with those for the Sun within a factor of 4.

Yamashita (1967) determined the excitation temperatures, the microturbulent velocities and the damping parameters separatly for some elements at the same ionization stage. For example, $\theta_{ex}=1.20\pm0.03$ (p.e.) for Fe I and $\theta_{ex}=1.23\pm0.03$ (p.e.) for Ti I, though in the case of Fe I, a sudden change in excitation temperature takes place near $\chi_1=3.0$ eV. The microturbulent velocities $V_{micro}$ were determined from the vertical shift $\log(c/2R_C V_D)$, where the correction for the thermal motion was made by equating the kinetic temperature to the excitation temperature. For example, $V_{micro}=3.3$ km/s for Ti I. In the case of Fe I, there is a tendency for the microturbulent velocity to decrease with the increase of the excitation potential, that is, the $V_{micro}$ values equal to 2.9 km/s, 2.4 km/s and 2.0 km/s for $\chi_1\leqq3.6$ eV, 3.6 eV$<\chi_1<$4.2 eV and $\chi_1\geqq4.2$ eV, respectively. The coincidence of damping parameters determined from different elements is fairly good. For example, $\log 2\alpha=-2.4$ for Fe I and $\log 2\alpha=-2.0$ for Ti I. The damping constants derived from the damping parameters are larger than the classical damping constant by a factor of two or three.

### III.2. The Results for $\psi$ UMa by the Method of Type 2

An absolute curve-of-growth analysis by the method of type 2 has been done in this paper, using the same data for Fe I and Ti I lines as Yamashita (1967).

The results for Fe I and Ti I are given in table 1 and 2, respectively. In these tables, the columns under the title of Case A give the results obtained by applying the method by rule, that is, the results determined simultaneously with $\log 2\alpha$ and $\log(c/2R_C V_D)$ values. On the other hand, the columns under the title of Case B give the results obtained by assuming $\log 2\alpha$ and $\log(c/2R_C V_D)$ values which were obtained by Yamashita (1967). In Case B, it is assumed that $R_C=1$. In this case, the $V_{micro}$ value for Fe I is taken to be 2.94 km/s which is the mean value of the $V_{micro}$ values for various elements obtained by Yamashita (1967). In these tables, the rows under the title of expres. (3) give the results obtained by adopting the $\log X_{ab}$ values given by the expression (3) according to Yamashita (1967). on the other hand, the rows under the title of expres. (1) give the results obtained by adopting the $\log X_{ab}$ values given by the expression (1). Moreover, the rows under the title of all give the results obtained from all the lines measured by Yamashita (1967), and the rows under the title of selected give the results obtained from the lines which have neither blended lines nor poor profiles. Equal weights are given to all the lines, except for the weight given according to the gradient of the curve-of-growth.

Except for Case B of Fe I, the $\theta_{ex}$ values agree with those obtained by Yamashita (1967) within probable errors. The $\theta_{ex}$ values for Case B of Fe I are larger than that obtained by Yamashita (1967) by 0.06~0.10. This seems to be caused by the adoption of the mean value for $V_{micro}$, which appears too large for Fe I. If, in fact, the different $V_{micro}$ values depending on the excitation potential are adopted according to Yamashita (1967), the results of $\theta_{ex}=1.22\pm0.0124$ (p.e.) and $\theta_{ex}=1.21\pm0.0172$ (p.e.) are obtained from all the lines and the selected lines, respectively, which agree with that by Yamashita (1967) within probable errors.

The $\log(c/2R_C V_D)$ values in Case A slightly differ from those obtained by Yamashita (1967), but the differences are not serious. Assuming that $R_C=1$ and that the kinetic temperature equals to 4170 K as were made by Yamashita (1967), we obtain that $V_{micro}$ =2.3 km/s~2.6 km/s for Fe I and $V_{micro}$ =3.3 km/s~4.1 km/s, which mostly agree with those by Yamashita (1967) within the error of ±0.3 km/s estimated by Yamashita (1967). The damping constants in Case A for Fe I are somewhat larger than that by Yamashita (1967). For example, we obtain $\Gamma=4.2\times10^8$ rad/s at 5000 Å from $\log 2\alpha=-1.9$ and from $\log(c/2R_C V_D)=4.75$, which is larger than $\Gamma=1.8\times10^8$ rad/s obtained by Yamashita (1967) by a factor of two.

## IV. Application to Carbon Star UHya

The method has also been applied to one of cool carbon stars U Hya in order to check the applicability of it to data with low accuracy.

### IV.1. Curve-of-Growth Analyses of Cool Carbon Stars

Model atmosphere analyses of cool carbon stars in visible regions of spectrum have not been done yet, for it is not known the nature of the atmospheres of carbon stars to a good approximation. The spectrum analyses of cool carbon stars in these spectral regions have been done by curve-of-growth analyses or by spectrum synthesis analyses.

Curve-of-growth analyses have been extensively done by Fujita and his co-workers. Especially Utsumi (1970) carried out absolute curve-of-growth analyses for 22 cool carbon stars in the spectral regions between about 4400 Å and 4500 Å and between about 4750 Å and 4900 Å which are relatively free from molecular absorption. Some lines are affected significantly by the blend effect of molecular bands even in these regions, so that the equivalent widths of these lines are seriously in error. When, in fact, Utsumi (1985)[9] reanalyzed these carbon stars by an absolute curve-of-growth method with new *fg* values, the abundances of rare-earth elements were reduced by about 0.5 dex due to the reductions of the equivalent widths after correction for the blend effect.

### IV.2. The Results for U Hya by the Method of Type 2

*U* Hya is one of cool carbon stars. It is classified as N2 according to the Harvard R-N system and classified as C6, 3 according to the C-classification system. Utsumi (1970) analyzed *U* Hya by an absolute curve-of-growth method. Utsumi (1985) reanalyzed this star also by the absolute curve-of-growth method, using new *gf* values and remeasured equivalent widths. In both of the analyses, he used the same theoretical curve-of-growth as adopted by Yamashita (1967) and in the method of type 2, that is, calculated by Hunger (1956), and he determined the atmospheric parameters such as excitation temperature, damping parameter and the Doppler velocity from Fe I and Ti I lines. In both of these analyses, the same values were obtained for excitation temperature and damping constant, that is, $\theta_{ex}=1.8$ and $\log 2\alpha=-3.0$, while the two values for Doppler velocity differ slightly, that is, $V_D=7.3$ km/s according to the old analysis and $V_D=6.7$ km/s according to the new analysis.

An absolute curve-of-growth analysis by the method of type 2 has been done in this paper, using the same data for Fe I and Ti I lines as Utsumi (1970). The numbers of lines are 35 and 34 for Fe I and for Ti I, respectively. According to Utsumi (1970), the $\log X_{ab}$ values given by the expression (1) were adopted as the abscissa of the curve-of-growth.

The results for Fe I and Ti I are given in table 3 and 4, respectively. In these tables, the columns under the title of Case A gives the results obtained by applying the method by rule. On the other hand, the columns under the title of Case B gives the results obtained by assuming the $\log 2\alpha$ and $\log (c/2R_C V_D)$ values which were obtained by Utsumi (1970). Equal weights are given to all lines, except for the weight given according to the gradient of the curve-of-growth.

Except for Case B of Ti I lines, the $\theta_{ex}$ value is smaller than that obtained by Utsumi (1970), that is, $\theta_{ex}=1.8$. Especially, the $\theta_{ex}$ value is extraordinary small in Case A. For example, $\theta_{ex}=0.88$ for Ti I, which corresponds to early G type star. The $\log (c/2R_C V_D)$ value is extraordinary large in Case A also. For example, $\log (c/2R_C V_D)=6.19$ for Ti I. Assuming that $R_C=1$ as was made by Utsumi (1970), we obtain $V=0.10$ km/s which is much smaller than the thermal velocity of the atmosphere of *U* Hya. All the lines, in this case, are on the damping part of the curve-of-growth. Furthermore, in Case A for Ti I, the damping parameter also is extraordinary small. That is, we obtain $\Gamma=4.0\times10^7$ rad/s at 5,000 Å from $\log 2\alpha=-1.5$ and from $V_D=0.10$ km/s, which is smaller even the classical radiation damping constant by a factor of two.

## V. Discussion

As examples of the degrees of scattering of lines in the empirical curve-of-growth, the curve-of-growths of Fe I for $\phi$ UMa and for *U* Hya are shown in figures 1 and 2, respectively. The corresponding correlations of Fe I lines for $\phi$ UMa and for *U* Hya between the horizontal shift $\Delta \log X$ and the excitation potential

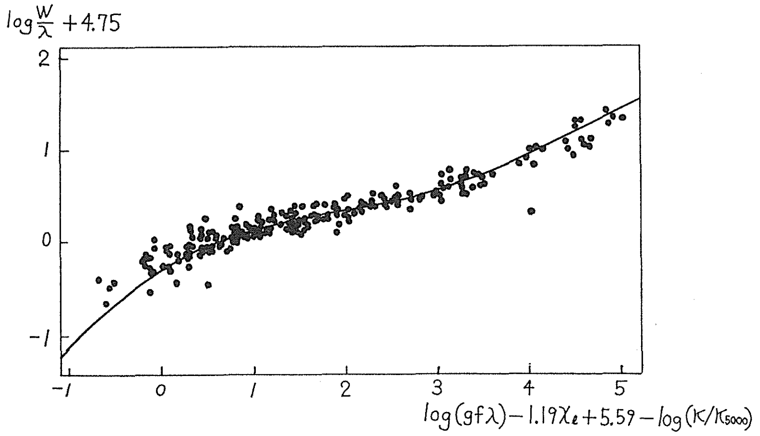


Fig. 1. Curve-of-growth of all the Fe I lines measured by Yamashita (1967) for $\phi$ UMa. The solid line is the theoretical curve with $\log 2\alpha = -1.9$ and $\log (c/2R_C V_D) = 4.75$, which is selected by the method of type 2. The filled circles are plotted by adopting the $\theta_{ex}$ value as 1.19.

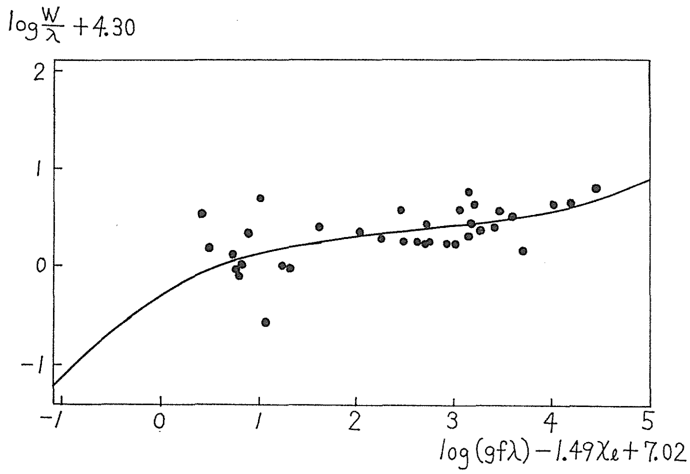


Fig. 2. Curve-of-growth of the Fe I lines measured by Utsumi (1970) for *U* Hya. The solid line is the theoretical curve with $\log 2\alpha = -3.0$ and $\log (c/2R_C V_D) = 4.3$, which was adopted by Utsumi (1970).

of the lower energy level $\chi_l$ are shown in figures 3 and 4, respectively.

The above results indicate that the method of type 2 can be applied to data with high accuracy such as those for $\phi$ UMa, while it cannot be applied by rule to data with low accuracy such as those for *U* Hya. Rather, in the latter case, it is better to give the $\log 2\alpha$ and $\log (c/2R_C V_D)$ values estimated by some means in advance of fitting an empirical curve-of-growth to a theoretical curve-of-growth

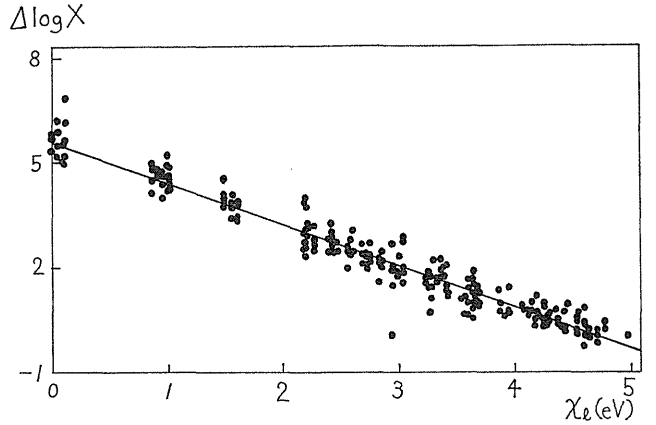


Fig. 3. Correlation between the horizontal shift $\Delta \log X$ and the excitation potential of the lower energy level $\chi_1$ for all the Fe I lines measured by Yamashita (1967) for $\phi$ UMa. The theoretical curve adopted is the same as that in figure 1. The solid line shows the least-squares solution: $\Delta \log X = -1.19\chi_1 + 5.59$.

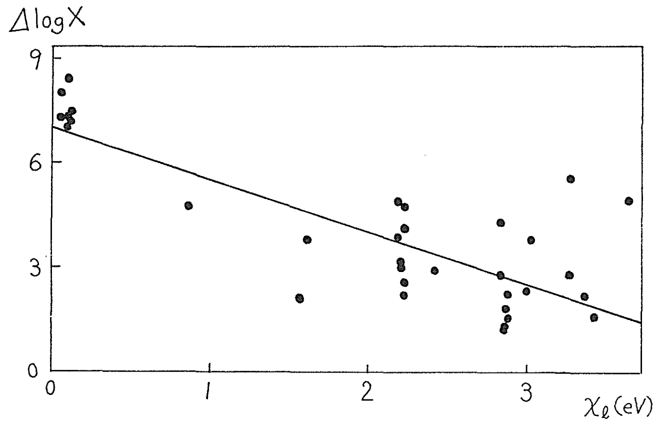


Fig. 4. Correlation between the horizontal shift $\Delta \log X$ and the excitation potential of the lower energy level $\chi_1$ for the Fe I lines measured by Utsumi (1970) for $U$ Hya. The theoretical curve adopted is the same as that in figure 2. The solid line shows the least-squares solution: $\Delta \log X = -1.49\chi_1 + 7.02$.

Table 1. Reciprocal excitation temperature of neutral iron for $\psi$ UMa determined by the method of type 2. Equal weights are given to all lines, except for the weight given according to the gradient of the curve-of-growth. The columns under Case A give the results obtained by applying the method by rule, and the columns under Case B give the results obtained by assuming $\log 2\alpha$ and $\log (c/2R_C V_D)$ values obtained by Yamashita (1967). The rows under the title of expres. (3) give the results obtained by adopting the $\log X_{ab}$ values given by the expression (3), and the rows under the title of expres. (1) give the results obtained by adopting the $\log X_{ab}$ values given by the expression (1).

| Adopted abscissa | Number of lines | Case A | | | Case B | | |
|---|---|---|---|---|---|---|---|
| | | Theoretical curve | | $\theta_{ex}\pm$p.e. | Theoretical curve | | $\theta_{ex}\pm$p.e. |
| | | $\log 2\alpha$ | $\log (c/2R_C V_D)$ | | $\log 2\alpha$ | $\log (c/2R_C V_D)$ | |
| expres. (3) | 271 all | −1.9 | 4.75 | 1.19 ±0.0099 | −2.4 | 4.68 | 1.27 ±0.0111 |
| | 153 selected | −1.9 | 4.77 | 1.20 ±0.0137 | −2.4 | 4.68 | 1.26 ±0.0161 |
| expres. (1) | 271 all | −1.8 | 4.73 | 1.19 ±0.095 | −2.4 | 4.68 | 1.30 ±0.0110 |
| | 153 selected | −1.8 | 4.76 | 1.20 ±0.0132 | −2.4 | 4.68 | 1.28 ±0.0160 |

Table 2. The same quantities for neutral titanium as those in table 1

| Adopted abscissa | Number of lines | Case A | | | Case B | | |
|---|---|---|---|---|---|---|---|
| | | Theoretical curve | | $\theta_{ex}\pm$p.e. | Theoretical curve | | $\theta_{ex}\pm$p.e. |
| | | $\log 2\alpha$ | $\log (c/2R_C V_D)$ | | $\log 2\alpha$ | $\log (c/2R_C V_D)$ | |
| expres. (3) | 63 all | −1.9 | 4.59 | 1.23 ±0.0281 | −2.0 | 4.63 | 1.26 ±0.0288 |
| | 27 selected | −2.4 | 4.63 | 1.29 ±0.0413 | −2.0 | 4.63 | 1.28 ±0.0419 |
| expres. (1) | 63 all | −1.8 | 4.55 | 1.21 ±0.0271 | −2.0 | 4.63 | 1.26 ±0.0289 |
| | 27 selected | −2.3 | 4.59 | 1.27 ±0.0380 | −2.0 | 4.63 | 1.28 ±0.0382 |

and determining the $\theta_{ex}$ value and the horizontal shift.

The following conclusions can be obtained from the above results.

1) Tables 1 and 2 indicate that the $\theta_{ex}$ values in the rows under the title of expres. (3) agree with those in the rows under the title of expres. (1) within probable errors. Furthermore, the probable errors in the rows under expres. (3) are larger than the corresponding errors in the rows under expres. (1). These

Table 3. Reciprocal excitation temperature of neutral iron for *U* Hya determined by the method of type 2. Equal weights are given to all lines, except for the weight given according to the gradient of the curve-of-growth. The columns under Case A give the results obtained by applying the method by rule, and the columns under Case B give the results obtained by assuming log $2\alpha$ and log $(c/2R_C V_D)$ values obtained by Yamashita (1967).

| Case A | | | Case B | | |
|---|---|---|---|---|---|
| Theoretical curve | | $\theta_{ex}$±p.e. | Theoretical curve | | $\theta_{ex}$±p.e. |
| log $2\alpha$ | log $(c/2R_C V_D)$ | | log $2\alpha$ | log $(c/2R_C V_D)$ | |
| −1.5 | 5.69 | 1.07±0.0850 | −3.0 | 4.3 | 1.49±0.1289 |

Table 4. The same quantities for neutral titanium as those in table 3

| Case A | | | Case B | | |
|---|---|---|---|---|---|
| Theoretical curve | | $\theta_{ex}$±p.e. | Theoretical curve | | $\theta_{ex}$±p.e. |
| log $2\alpha$ | log $(c/2R_C V_D)$ | | log $2\alpha$ | log $(c/2R_C V_D)$ | |
| −1.5 | 6.19 | 0.88±0.1033 | −3.0 | 4.3 | 1.92±0.1750 |

results may mean that the variation of the continuous absorption coefficient is insignificant in comparison with the errors of data of absorption lines such as those of oscillator strengths and equivalent widths.

2) Some of the differences in the results between this and Yamashita (1967) may be due to the difference in the procedure of the curve-of-growth analysis. In the fitting of an empirical curve-of-growth to a theoretical curve, Yamashita (1967) classed lines into several groups with small ranges of excitation potentials, while in this study each line is treated separately. Especially, the sudden cange in excitation temperature near $\chi_1=3.0$ eV does not take place in this analysis, as is shown in figure 3 This may means that the sudden change is not the real phenomenon such as the screening effect of the continuum suggested by Yamashita (1967).

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## Appendix. List of the Program "COG 0"

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10 REM Determination of Δθ and <ΔlogA> for Single Species
20 REM The Selected Curve Minimizes the Value for the Sum
30 REM of the Squares of Differences along the Abscissa
40 REM for a group of vertical shift in a given range
50 REM The Gradient of the Curve Is Taken into Account for Weight
60 DEFINT I-K:DEFDBL V:WIDTH 80,25
70 DIM LAMDA(320),POTEN(320),LGW(320),LGX(320),DKAI(320)
80 DIM W(320),IW(320),DX(320,50),DDX(320,50),LGXA(320),AAA(50)
90 DIM ALFA(8),Y(11),TATE(12),TTATE(12,8),A(11,2)
100 INPUT "If neutral push N or ion push I";A$
110 DIM THETA(50),AA(50),PETHETA(50),PEAA(50)
120 IF ANS$="N" GOTO 360
130 INPUT "log(2α)";AALFA
140 OPEN "2:DATA1" FOR INPUT AS #1
150 II=0
160 IF EOF(1) THEN CLOSE #1:GOTO 200
170 INPUT #1,ALFA(II)
180 FOR J=0 TO 12:INPUT #1,TTATE(J,II):NEXT J
190 II=II+1:GOTO 160
200 FOR II=0 TO 8
210 IF AALFA>=ALFA(II) GOTO 230
220 NEXT II
230 S1=2*(AALFA-ALFA(II)):S2=S1-.5:S3=S1*(S1-1)
240 IF II<8 GOTO 300
250 FOR J=0 TO 12
260 TATE(J)=TTATE(J,8)+(TTATE(J,7)-TTATE(J,8))*S1
270 TATE(J)=TATE(J)+(TTATE(J,6)+TTATE(J,8)-2*TTATE(J,7))*S3/4
280 NEXT J
290 GOTO 350
300 FOR J=0 TO 12
310 TATE(J)=(TTATE(J,II)+TTATE(J,II-1))/2
320 TATE(J)=TATE(J)+(TTATE(J,II-1)-TTATE(J,II))*S2
330 TATE(J)=TATE(J)+(TTATE(J,II-2)+TTATE(J,II+1)-TTATE(J,II-1)-TTATE(J,II))*S3/4
340 NEXT J
350 GOSUB *HENKAX
360 INPUT "min. and max. and step value of log(c/2RV)";VMIN,VMAX,VD
370 DKAI1=0:DKAI2=0
380 IF ANS$<>"" GOTO 490
390 I=0:JJ=0:IWW=0
400 OPEN "2:DATA  " FOR INPUT AS #1
410 IF EOF(1) THEN CLOSE #1:GOTO 480
420 INPUT #1,LAMDA(I),IND,POTEN(I),LGW(I),LGX(I),IW(I)
430 IF A$="I" GOTO 450
440 DKAI(I)=7.87-POTEN(I):GOTO 460
450 DKAI(I)=-POTEN(I)
460 IWW=IWW+IW(I)
470 I=I+1:JJ=JJ+1:GOTO 410
480 I=I-1
490 V=VMIN:IN=1
500 WHILE V<VMAX+.0001#
510 PX=0:PXY=0:WW=0:DKAI1=0:DKAI2=0
520 FOR J=0 TO I
530 YY=LGW(J)+V:GOSUB *KEISAN:GOSUB *KEISAN1
540 DX(J,IN)=X-V-.052-LGX(J):W(J)=IW(J)/DC:WW=WW+W(J)
550 NEXT J
560 WR=IWW/WW:WW=0
570 FOR J=0 TO I:W(J)=W(J)*WR:WW=WW+W(J):NEXT J
580 FOR J=0 TO I
590 DKAI1=DKAI1+W(J)*DKAI(J):DKAI2=DKAI2+W(J)*DKAI(J)^2
600 PX=PX+W(J)*DX(J,IN):PXY=PXY+W(J)*DX(J,IN)*DKAI(J)
610 NEXT J
620 D=DKAI2*WW-DKAI1^2
630 THETA(IN)=(PXY*WW-DKAI1*PX)/D:AAA(IN)=(DKAI2*PX-DKAI1*PXY)/D
640 PP=0
650 FOR J=0 TO I
660 DDX(J,IN)=DX(J,IN)-THETA(IN)*DKAI(J)-AAA(IN):PP=PP+W(J)*DDX(J,IN)^2
670 NEXT J
680 PETHETA(IN)=.67449*SQR(WW*PP/(JJ-2)/D):PEAA(IN)=.67449*SQR(DKAI2*PP/(JJ-2)/D)
690 V=V+VD:IN=IN+1
700 WEND
710 IN=IN-1
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720 MINPET=PETHETA(1):IM=1
730 FOR J=2 TO IN
740 IF PETHETA(J)<MINPET THEN MINPET=PETHETA(J):IM=J
750 NEXT J
760 V=VMIN+VD*(IM-1):AA=AAA(IM)
770 INPUT "Critical Difference : If CD<0, data are not listed";CD
780 CLS 3
790 LPRINT "No. with min. prob. error";IM,"(max. No.)";IN:LPRINT
800 PRINT "No. with min. prob. error";IM,"(max. No.)";IN:PRINT
810 LPRINT "log (2α) = ";AALFA,"log (c/2RV) = ";V,"Number of Lines = ";JJ:LPRINT
820 PRINT "log (2α) = ";AALFA,"log (c/2RV) = ";V,"Number of Lines = ";JJ:PRINT
830 PRINT "Δθ  =";THETA(IM),"Probable Error=";PETHETA(IM)
840 LPRINT "Δθ  =";THETA(IM),"Probable Error=";PETHETA(IM)
850 PRINT " [A] = ";AA,"Probable Error=";PEAA(IM):PRINT
860 LPRINT " [A] = ";AA,"Probable Error=";PEAA(IM):LPRINT
870 IF CD<0 GOTO 960
880 LPRINT :LPRINT "Critical Difference = ";CD:LPRINT
890 PRINT "Wavelength";SPC(5);"Log(W/λ)     LogX";SPC(11);"Difference":PRINT
900 LPRINT "Wavelength";SPC(5);"Log(W/λ)     LogX";SPC(11);"Difference":LPRINT
910 FOR J=0 TO I
920 IF ABS(DDX(J,IM))<CD GOTO 950
930 PRINT LAMDA(J);SPC(7);LGW(J);SPC(7);LGX(J);SPC(7);DDX(J,IM)
940 LPRINT LAMDA(J);SPC(7);LGW(J);SPC(7);LGX(J);SPC(7);DDX(J,IM)
950 NEXT J
960 ANS$=INKEY$
970 IF ANS$="" GOTO 960
980 CLS 3
990 INPUT "Graphic (Relation [X] vs Δχ) Y or N";ANS$
1000 IF ANS$="N" GOTO 1040
1010 GOSUB *GRAPH1
1020 ANS$=INKEY$
1030 IF ANS$="" GOTO 1020
1040 CLS 3:CONSOLE 0,25,1,0:LOCATE 0,0,1
1050 INPUT "Graphic(Curve of Growth) Y or N";ANS$
1060 IF ANS$="N" GOTO 1110
1070 GOSUB *GRAPH2
1080 ANS$=INKEY$
1090 IF ANS$="" GOTO 1080
1100 CLS 3:CONSOLE 0,25,1,0:LOCATE 0,0,1
1110 INPUT "Repeat Y or N";ANS$
1120 IF ANS$="N" GOTO 1150
1130 INPUT "Repeat with new log(2α)  Y or N";ANS$
1140 PRINT :LPRINT :GOTO 120
1150 END
1160 *HENKAX
1170 REM LogC=A(I,0)*Y^2+A(I,1)*Y+A(I,2)        Y=log(W/(2·R·Δλ))
1180 IF ANS$<>"" GOTO 1200
1190 DIM E(10),F(10),G(10)
1200 Z=-1
1210 D=(TATE(0)-TATE(1))*(TATE(1)-TATE(2))*(TATE(2)-TATE(0))
1220 E(0)=(TATE(1)-(TATE(0)+TATE(2))/2)/D
1230 F(0)=((TATE(0)^2+TATE(2)^2)/2-TATE(1)^2)/D
1240 G(0)=Z-(TATE(0)*(2*TATE(1)*(TATE(0)-TATE(1))+TATE(2)*(TATE(2)-TATE(0))))/(2*D)
1250 A(0,0)=E(0):A(0,1)=F(0):A(0,2)=G(0)
1260 FOR II=1 TO 10
1270 Z=Z+.5
1280 D=(TATE(II)-TATE(II+1))*(TATE(II+1)-TATE(II+2))*(TATE(II+2)-TATE(II))
1290 E(II)=(TATE(II+1)-(TATE(II)+TATE(II+2))/2)/D
1300 F(II)=((TATE(II)^2+TATE(II+2)^2)/2-TATE(II+1)^2)/D
1310 G(II)=TATE(II)*(2*TATE(II+1)*(TATE(II)-TATE(II+1))+TATE(II+2)*(TATE(II+2)-TATE(II)))
1320 G(II)=Z-G(II)/(2*D)
1330 A(II,0)=(E(II-1)+E(II))/2:A(II,1)=(F(II-1)+F(II))/2:A(II,2)=(G(II-1)+G(II))/2
1340 NEXT II
1350 FOR II=0 TO 11:Y(II)=TATE(II+1):NEXT II
1360 A(11,0)=E(10):A(11,1)=F(10):A(11,2)=G(10)
1370 RETURN
1380 *KEISAN
1390 REM LogC is calculated from Log(W/(2·R·Δλ))
1400 IF YY=TATE(0) GOTO 1480
1410 FOR K=0 TO 11
1420 IF YY<=Y(K) GOTO 1450
1430 NEXT K
1440 X=-.1437+2*YY-AALFA+LOG(1+SQR(1+2.4674*(10^AALFA/10^YY)^2))/LOG(10):GOTO 1490
1450 X=A(K,0)
1460 FOR II=1 TO 2:X=X*YY+A(K,II):NEXT II
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1470 RETURN
1480 X=9*(YY+10.052)/(TATE(0)+10.052)-10
1490 RETURN
1500 *GRAPH1
1510 CLS 3:SCREEN 2,0:COLOR 0:CONSOLE 0,25,0,0
1520 DKAIMAX=0:DXMIN=100:DXMAX=-100
1530 FOR J=0 TO I
1540 IF IW(J)=0 GOTO 1580
1550 IF ABS(DKAI(J))>DKAIMAX THEN DKAIMAX=ABS(DKAI(J))
1560 IF DX(J,IM)<DXMIN THEN DXMIN=DX(J,IM)
1570 IF DX(J,IM)>DXMAX THEN DXMAX=DX(J,IM)
1580 NEXT J
1590 PRINT "Max(ΔlogX)=";DXMAX,"Min(ΔlogX)=";DXMIN
1600 INPUT "Max.Graduation, Min.Graduation, Interval of Graduation";MAXGR,MINGR,DGR
1610 MSO=320/(MAXGR-MINGR):MSA=590/DKAIMAX:CLS 3
1620 LINE(40,345)-(635,345):LINE(40,20)-(40,345)
1630 IGO=INT((MAXGR-MINGR)/DGR+.1)
1640 FOR J=0 TO IGO
1650 LINE(37,25+CINT(MSO*J*DGR))-(40,25+CINT(MSO*J*DGR))
1660 NEXT J
1670 IGA=INT(DKAIMAX)
1680 FOR J=0 TO IGA:LINE(40+CINT(MSA*J),345)-(40+CINT(MSA*J),348):NEXT J
1690 LOCATE 0,25,0
1700 IF A$="N" GOTO 1730
1710 FOR J=0 TO IGA:PRINT TAB(4+INT(J*MSA/8));:PRINT -J;:NEXT J
1720 GOTO 1740
1730 FOR J=0 TO IGA:PRINT TAB(4+INT(J*MSA/8));:PRINT J;:NEXT J
1740 FOR J=0 TO IGO
1750 LOCATE 0,2+INT(MSO*J*DGR/16),0:PRINT MAXGR-J*DGR;
1760 NEXT J
1770 LOCATE 0,1,0:PRINT "ΔlogX";
1780 IF A$="N" THEN LOCATE 18,24,0:PRINT "Δχ" ELSE LOCATE 18,24,0:PRINT "-χ"
1790 FOR J=0 TO I
1800 IF IW(J)=0 GOTO 1850
1810 IF A$="N" GOTO 1830
1820 IPX=40+CINT(-MSA*DKAI(J)):IPY=25+CINT(MSO*(MAXGR-DX(J,IM))):GOTO 1840
1830 IPX=40+CINT(MSA*DKAI(J)):IPY=25+CINT(MSO*(MAXGR-DX(J,IM)))
1840 CIRCLE(IPX,IPY),3:PAINT(IPX,IPY)
1850 NEXT J
1860 IF A$="I" GOTO 1930
1870 IPX1=630:IPY1=25+CINT(MSO*(MAXGR-AA-THETA(IM)*DKAIMAX))
1880 IF AA>MAXGR+.5/MSO THEN IPY2=25:IPX2=40+CINT(MSA*(MAXGR-AA)/THETA(IM)):GOTO 1910
1890 IF AA<MINGR-.5/MSO THEN IPY2=345:IPX2=40+CINT(MSA*(MINGR-AA)/THETA(IM)):GOTO 1910
1900 IPX2=40:IPY2=25+CINT(MSO*(MAXGR-AA))
1910 LINE(IPX1,IPY1)-(IPX2,IPY2)
1920 RETURN
1930 IPX1=630:IPY1=25+CINT(MSO*(MAXGR-AA+THETA(IM)*DKAIMAX))
1940 IF AA>MAXGR+.5/MSO THEN IPY2=25:IPX2=40-CINT(MSA*(MAXGR-AA)/THETA(IM)):GOTO 1970
1950 IF AA<MINGR-.5/MSO THEN IPY2=345:IPX2=40-CINT(MSA*(MINGR-AA)/THETA(IM)):GOTO 1970
1960 IPX2=40:IPY2=25+CINT(MSO*(MAXGR-AA))
1970 LINE(IPX1,IPY1)-(IPX2,IPY2)
1980 RETURN
1990 *GRAPH2
2000 CLS 3:SCREEN 2,0:COLOR 0:CONSOLE 0,25,0,0
2010 FOR J=0 TO I:LGXA(J)=LGX(J)+THETA(IM)*DKAI(J)+AA+V+.052:NEXT J
2020 LINE(20,360)-(630,360)
2030 LINE(20,20)-(20,360)
2040 FOR J=0 TO 3:LINE(17,20+J*100)-(20,20+J*100):NEXT J
2050 FOR J=0 TO 6:LINE(30+J*100,360)-(30+J*100,363):NEXT J
2060 LOCATE 3,22,0:PRINT TAB(69);:PRINT "LogC"
2070 LOCATE 2,25,0:FOR J=0 TO 6:PRINT TAB(2+INT(J*12.5));:PRINT J-1;:NEXT J
2080 LOCATE 3,0,0:PRINT "Log(W/(2RΔλ))";SPC(18);"log2α = ";AALFA;SPC(8);"log(c/2RV) = ";V
2090 FOR J=0 TO 3:LOCATE 0,1+INT(6.25*J),0:PRINT 2-J:NEXT J
2100 PPY=2
2110 YY=PPY:GOSUB *KEISAN
2120 IF X>5 THEN PPY=PPY-.1:GOTO 2110
2130 IPX1=CINT(100*X)+130:IPY1=220-CINT(100*PPY)
2140 PPY=PPY-.1:YY=PPY:GOSUB *KEISAN
2150 IF PPY<-1.4 OR X<-1.21 GOTO 2190
2160 IPX2=CINT(100*X)+130:IPY2=220-CINT(100*PPY)
2170 LINE(IPX1,IPY1)-(IPX2,IPY2)
2180 IPX1=IPX2:IPY1=IPY2:GOTO 2140
2190 FOR J=0 TO I
2200 IPX=CINT(100*LGXA(J)):IPY=CINT(100*(LGW(J)+V))
2210 IF IPX>500 OR IPY>200 GOTO 2250
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2220 IF IPX<-110 OR IPY<-140 GOTO 2250
2230 IPX=IPX+130:IPY=220-IPY
2240 CIRCLE(IPX,IPY),3:PAINT(IPX,IPY)
2250 NEXT J
2260 RETURN
2270 *KEISAN1
2280 REM Gradient of the curve is calculated from log(W/(2・R・Δλ))
2290 IF YY<TATE(0) THEN DC=9/(TATE(0)+10.052):RETURN
2300 FOR K=0 TO 11
2310 IF YY<=Y(K) GOTO 2340
2320 NEXT K
2330 DC=2:RETURN
2340 DC=2*A(K,0)*YY+A(K,1):RETURN
```